PHYS1110D – Engineering Physics: Mechanics and Thermodynamics

Tutorial Problems for Week 2: Basic Concepts of Functions and Derivatives

**Problem 1 – Translation of Function Graphs (Preparing to Learn Waves)**

1. Sketch the curves of the following functions on the same graph:
2. How does the curve change when is replaced by ( where is a positive constant?

**Solution:**



1. : Moving towards left; : Moving towards right.

This conclusion can be obtained by fixing

We see that for the same argument of the function , the required becomes

Corresponding to smaller/larger .

**Problem 2 – Derivatives and Extremum of Functions**

Given a function , where and are some constants.

1. Find and .
2. At which value of does the function reaches its extremum? Is it a local maximum or minimum?

**Solution:**

1. By direct differentiation, we can find

*Remark*: We can think as the displacement of an object. Then and are velocity and acceleration respectively. Therefore, the above describes the motion of an object under constant acceleration.

1. The function assumes its local extremum when

To check whether it is a local maximum or minimum, we calculate

Thus, it is a local maximum.

**Problem 3 – Second Derivative and Convexity of Functions**

Complete the below table with .

* In the first and second row (except the grey cells), determine if the derivative is positive or negative;
* In the last row (except the grey cells), draw the tendency (monotonicity and convexity) of the curve;
* In the grey cells, fill in the corresponding values.

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Then, sketch the curve of by using the above table.

*Hints*:

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**Solution:**

We first calculate the derivatives

Hence:

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|  |  | 0 |  | 4 |  | 2 |  | 0 |  |

Sketch of the curve:

*Remark:* The point is special: *vanishes* at this point, but takes *different sign* on its two sides. It is then called an **inflection point**of the function , for the function *changes its convexity* there.

**Problem 4 – Application of Differentiation in Kinetics**

The displacement of a ball is recorded as

where is in meters, and is in seconds.

Find the velocity and acceleration of the ball for .

**Solution:**

By definition of velocity and acceleration

**Problem 5 – Preparing to Learn Integration**

1. As a lemma, prove that

*Hint*:

1. Simplify the expression

When:

* 1. .

*Hint*:

* 1. .

*Hint*: Use the result in question 1)

**Solution:**

Therefore

*Remark:* The last two questions in fact explain how to do integration of polynomials.

Mathematically, integration of a function from to is defined as

You will learn its geometrical meaning in the lectures. Then, taking the limit , we obtain